



Two-loop $\mathcal{O}(\alpha_s y^2)$ and $\mathcal{O}(y^4)$ MSSM corrections to the pole mass of the b -quark

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Abstract

The Letter is devoted to the calculation of the two-loop $\mathcal{O}(\alpha_s y^2)$ and $\mathcal{O}(y^4)$ MSSM corrections to the relation between the pole mass of the b -quark and its running mass in the $\overline{\text{DR}}$ scheme. To evaluate the needed diagrams the large mass expansion procedure is used. The obtained contributions are negative in most of the regions of the parameter space and partly compensate the positive $\mathcal{O}(\alpha_s^2)$ contribution calculated earlier.

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1. Introduction

The minimal supersymmetric standard model (MSSM) became an underlying framework for both the theoretical and experimental research in supersymmetric phenomenology. During recent years considerable progress has been achieved in calculation of various processes involving supersymmetric particles and the parameter space of the MSSM is severely con-

strained by numerous phenomenological fits [1–5]. These fits require precision calculations of radiative corrections due to virtual supersymmetric particles. Due to the complexity of the MSSM Lagrangian these calculations are very lengthy and presuppose the creation of the proper computer codes.

In this Letter, we calculate two-loop radiative corrections to the pole mass of the b -quark within the MSSM. The one-loop correction has been calculated in [6] and is very essential. It reduces considerably the value of the running quark mass extracted from the pole mass. It should be noted that the leading $\mathcal{O}(\alpha_s)$ contribution from stop-gluino loop which is positive

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is reduced significantly by a large negative correction from stop-chargino loop that is proportional to the Yukawa couplings of heavy quarks (y_t , y_b), $\tan \beta$, and the Higgs mixing parameter μ . For large $\tan \beta$ one has to take into account not only y_t but also y_b since it becomes large as well. The two-loop correction of the order of $\mathcal{O}(\alpha_s^2)$ has been calculated in [7]. It was found that this correction was also positive and of the same order of magnitude as the one-loop MSSM contribution.

The aim of this Letter is to present our calculations of the corrections proportional to the Yukawa couplings. We calculate the two-loop $\mathcal{O}(\alpha_s y^2)$ and $\mathcal{O}(y^4)$ MSSM corrections to the relation between the pole mass of the b -quark and its running mass in the $\overline{\text{DR}}$ scheme [8]. The $\overline{\text{DR}}$ scheme corresponds to a theory regularized by dimensional reduction (from space-time dimension $d = 4$ to $d = 4 - 2\varepsilon$) and renormalized minimally. This allows one to keep supersymmetry unbroken and use the computational advantages of dimensional regularization.

To evaluate the needed diagrams we made use of the large mass expansion procedure [9] and restricted ourselves to the terms up to $\mathcal{O}(m_b^2/M_{\text{hard}}^2)$, where M_{hard} stands for all mass scales involved in the problem that are much larger than m_b . Since the total number of diagrams exceeds 1000, we are not able to present the calculation in a compact form, so we provide a numerical demonstration of the results. The final result is also available upon request in a form of C++ code.¹ As expected, the calculated corrections are negative in most of the regions of the parameter space of the MSSM and partly compensate those proportional to $\mathcal{O}(\alpha_s^2)$. By absolute value the “leading” $\mathcal{O}(\alpha_s^2 + \alpha_s y^2 + y^4)$ two-loop corrections are of the order of 30 to 40 percent of the one-loop ones.

2. Pole mass of the b -quark

The pole mass of a particle is defined as a real part of the complex pole of the resummed propagator (we discuss only perturbative effects). The full connected

propagator of a quark can be written as²

$$\frac{i}{\hat{p} - m - \Sigma(\hat{p}, m_i)}, \quad (1)$$

where

$$\Sigma(\hat{p}, m_i) = \hat{p} \Sigma_V(p^2, m_i^2) + \hat{p} \gamma_5 \Sigma_A(p^2, m_i^2) + m \Sigma_S(p^2, m_i), \quad (2)$$

is the self-energy of the quark, so the pole mass M_{pole} satisfies the following equation:

$$(1 + \Sigma_V(M_{\text{pole}}^2, m_i^2))^2 M_{\text{pole}}^2 - \Sigma_A^2(M_{\text{pole}}^2, m_i^2) M_{\text{pole}}^2 - m^2 (1 - \Sigma_S(M_{\text{pole}}^2, m_i^2))^2 = 0. \quad (3)$$

Solving this equation perturbatively, one gets

$$\frac{M_{\text{pole}} - m}{m} = \alpha M^{(1)} + \alpha^2 M^{(2)}, \quad \text{where} \quad (4)$$

$$M^{(1)} = \Sigma_V^{(1)}(m^2, m_i^2) + \Sigma_S^{(1)}(m^2, m_i), \quad (5)$$

$$\begin{aligned} M^{(2)} = & \Sigma_V^{(2)}(m^2, m_i^2) + \Sigma_S^{(2)}(m^2, m_i) \\ & + \frac{1}{2} \Sigma_A^{(1)2}(m^2, m_i^2) \\ & + M^{(1)} \left(\Sigma_V^{(1)}(m^2, m_i^2) \right. \\ & + 2m^2 \frac{\partial}{\partial p^2} (\Sigma_V^{(1)}(p^2, m_i^2) \\ & \left. + \Sigma_S^{(1)}(p^2, m_i)) \right) \Big|_{p^2=m^2}, \end{aligned} \quad (6)$$

and α stands for all couplings of the theory.

Using Eq. (4), one calculates the relation between pole and running masses of the b -quark (depending on the prescription used for evaluation of $\Sigma^{(i)}$, mass parameter m in (4) may correspond to bare or renormalized mass, so one obtains the relation between pole and bare masses or between pole and running masses).

We use regularization by dimensional reduction $\overline{\text{DR}}$, a modification of the conventional dimensional regularization, originally proposed in [8], and the same renormalization prescription as in [7,10]. Therefore, we calculate

$$\frac{\Delta m_b}{m_b} \equiv \frac{M_b^{\text{pole}} - m_b^{\text{run}}}{m_b^{\text{run}}}, \quad (7)$$

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² The CKM matrix is supposed to be diagonal.

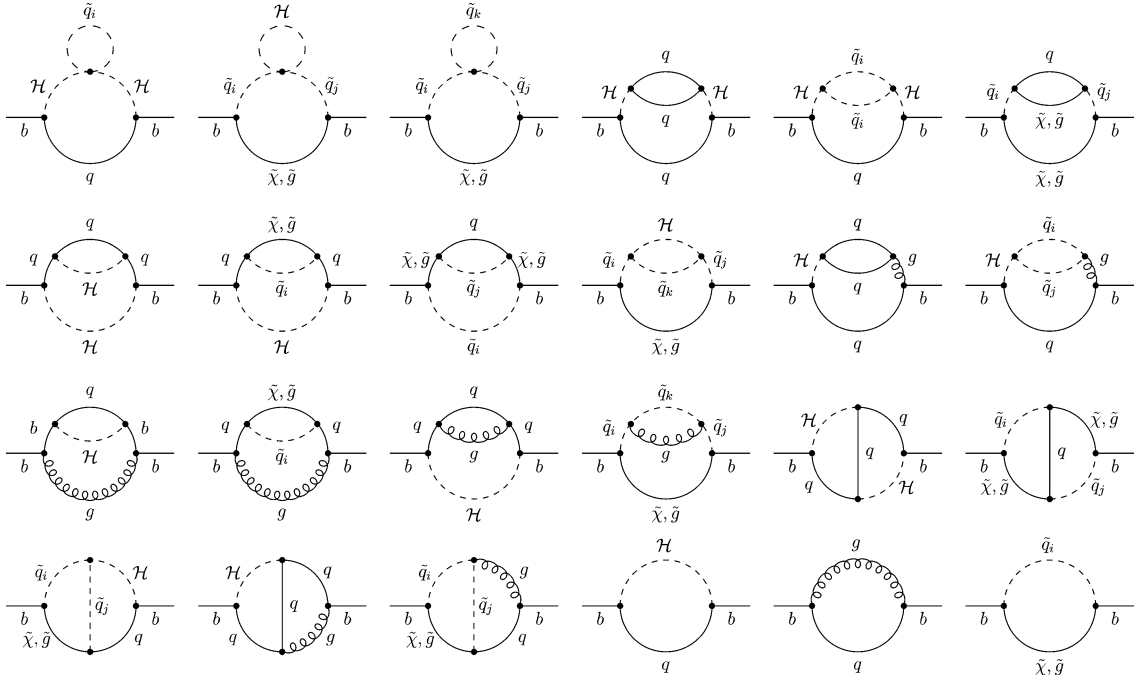


Fig. 1. Feynman diagrams contributing to quark self-energies. Here q , \tilde{q}_i , g , \tilde{g} denote quark, squark, gluon, and gluino fields, respectively, $\mathcal{H} = \{h_0, H_0, A_0, G_0, H^+, G^+\}$ corresponds to various Higgs fields and $\tilde{\chi} = \{\tilde{\chi}_0, \tilde{\chi}^\pm\}$ are higgsino states. Two-loop diagrams of the order $\mathcal{O}(\alpha_s^2)$ are not presented here (see, e.g., [7]).

where M_b^{pole} stands for the pole mass of the b -quark and $m_b^{\text{run}} \equiv m_b(\bar{\mu})$ corresponds to the running $\overline{\text{DR}}$ mass of the b -quark at the scale $\bar{\mu}$.

To evaluate this quantity, we have to calculate more than 1000 two-loop propagator-type diagrams (see Fig. 1). In order to simplify our calculations, we used the 't Hooft–Feynman gauge ($\xi_W = 1$, $\xi_Z = 1$). We also neglected mixing in the chargino and neutralino sectors, so only higgsino states were taken into account. This implies that all higgsinos have the same mass which is equal to the Higgs mixing parameter μ .

Due to the presence of many different mass scales, the problem of exact evaluation of two-loop diagrams is rather complex. Exploiting the fact that

$$m_b \ll m_t, m_{h_0}, m_{H_0}, m_{H^+}, m_{\tilde{q}}, m_{\tilde{\chi}^0}, m_{\tilde{\chi}^\pm}, m_{G_0}, m_{G^+}, \quad (8)$$

where m_b and m_t are masses of bottom and top quarks, respectively, m_{h_0} , m_{H_0} , m_{H^+} are masses of the Higgs bosons, $m_{\tilde{\chi}^0}$ is the neutralino mass, $m_{\tilde{\chi}^\pm}$ is the chargino mass, $m_{\tilde{q}}$ are masses of different squarks,

m_{G_0} , m_{G^+} are masses of pseudo-Goldstone bosons, we use the method of large mass expansion [9] to reduce the evaluation of multi-scale two-loop diagrams to the calculation of two-loop vacuum integrals and products of one-loop on-shell propagator-type diagrams and one-loop bubble integrals. We include only terms up to $\mathcal{O}(m_b^2/M_{\text{hard}}^2)$, where M_{hard} denotes any mass of the right-hand side of (8).

A general Feynman diagram F_Γ which depends on the large masses M_1, M_2, \dots , small masses m_1, m_2, \dots , and small external momenta p_1, p_2, \dots can be expanded as follows [9]:

$$F_\Gamma(p_1, \dots, M_1, \dots, m_1, \dots) = \sum_\gamma F_{\Gamma/\gamma}(p, m) \mathcal{M}_\gamma(p_\gamma, m) F_\gamma(M, m, p_\gamma), \quad (9)$$

where the operator $\mathcal{M}_\gamma(p_\gamma, m)$ performs Taylor expansion in small external (with respect to the subgraph γ) momenta and masses. The sum runs over all asymptotically irreducible subgraphs of the original graph Γ .

For calculation of a two-loop propagator diagram that does not have cuts composed of two light lines up

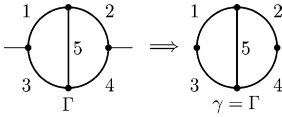


Fig. 2. The diagram has no cuts composed of two light lines; therefore, the naive expansion can be used.

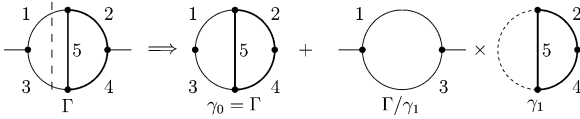


Fig. 3. The diagram has one cut composed of two light lines, so non-trivial subgraph γ_1 has to be taken into account.

to $\mathcal{O}(p^2/M^2)$ one can use naive expansion, when the sum in (9) runs over only the trivial subgraph $\gamma = \Gamma$ (see Fig. 2). In the case of a diagram that has such a cut, a non-trivial subgraph γ_1 has to be taken into account³ (see Fig. 3).

Two-loop vacuum integrals can be recursively reduced to a master-integral [12] by integration by parts method [13].

Our calculation can be performed in a semi-automatic way. First, FeynArts [14] is used to generate the diagrams. Then the contribution of individual diagrams to relation (4) is evaluated by means of the C++ program TwoLoop, based on GiNaC [15]. TwoLoop [16] performs large mass expansion according to (9), uses recurrence relations of [12] to reduce two-loop vacuum integrals to the master integral, and performs expansion in ε . The results of this calculation were analytically cross-checked using FORM program based on the ON-SHELL2 package [17].

After evaluation of two-loop diagrams one should substitute them into (4) and perform renormalization in order to express the correction (7) in terms of the running $\overline{\text{DR}}$ parameters. Counter-terms were calculated and checked in the same way as in [7], e.g., our renormalization constants for the gauge and Yukawa couplings reproduce the well-known β -functions [18].

³ Asymptotic expansion of such a diagram was discussed in detail in [11].

3. Numerical results

The analytical result of our calculation is very complicated due to the presence of a large number of masses, and no phenomenologically acceptable limit seems to exist. Therefore, we present here the numerical analysis of our results.

While a detailed scan over the more-than-hundred-dimensional parameter space of MSSM is clearly not practicable, even a sampling of the three- (four-) dimensional CMSSM parameter space of m_0 , $m_{1/2}$ and A_0 ($\tan\beta$) is beyond the present capabilities of phenomenological studies, especially when one tries to simulate experimental signatures of supersymmetric particles within a detector. For this reason, one often resorts to specific benchmark scenarios, i.e., one studies only specific parameter points or at best samples the one-dimensional parameter space [19,20].

Numerical values of running SUSY parameters at the M_Z scale have been calculated as a function of CMSSM parameters with the program TwoLoop [16] after interfacing it to a slightly modified version of SOFTSUSY [21] in the framework of the mSUGRA supersymmetry breaking scenario. We only consider $\mu > 0$ and large values of $\tan\beta$, since small $\tan\beta$ and negative μ seem to be excluded by experimental data [2].

Fig. 4 shows $m_{1/2}$ -dependence of the $\mathcal{O}(\alpha_s^2)$ [7] and $\mathcal{O}(\alpha_s y^2 + y^4)$ MSSM contributions to (7). For comparison, the full one-loop MSSM correction was plotted. Numerical value of the $\mathcal{O}(\alpha_s^2)$ contribution is comparable with the full one-loop result, so neglecting other two-loop MSSM corrections leads to a contradiction with the perturbation theory. The $\mathcal{O}(\alpha_s y^2 + y^4)$ contribution is negative in a wide range of the CMSSM parameter space and reduces the $\mathcal{O}(\alpha_s^2)$ contribution by approximately 30–80%. As a consequence the $\mathcal{O}(\alpha_s^2 + \alpha_s y^2 + y^4)$ two-loop correction appears to be less than 40% of the one-loop contribution and, therefore, can be used in phenomenological studies of CMSSM.

Varying A_0 in a wide range from -1000 to 1000 GeV does not change the behaviour of the $\mathcal{O}(\alpha_s^2 + \alpha_s y^2 + y^4)$ correction significantly, as one can see from Fig. 4. To make this fact more clear, several curves corresponding to different values of A_0 were plotted in Fig. 5.

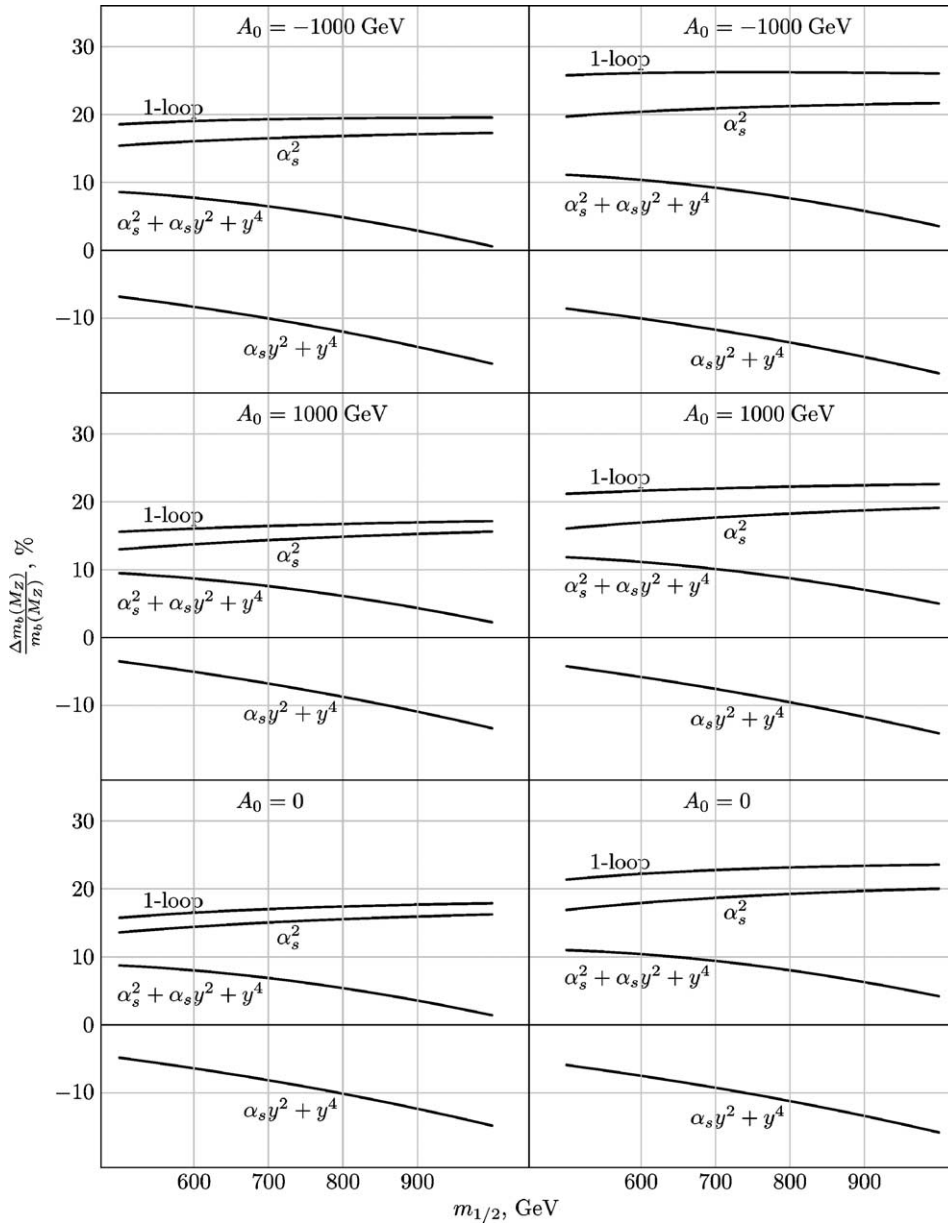


Fig. 4. Different MSSM contributions to $\Delta m_b(M_Z)/m_b(M_Z)$. Here $m_0 = 1000$ GeV, the left column corresponds to $\tan \beta = 35$, and the right column corresponds to $\tan \beta = 50$.

In Fig. 6, we present the dependence of different two-loop MSSM contributions on m_0 . For $m_{1/2} \gtrsim 500$ GeV the change of m_0 from 400 to 1000 GeV yields the change of the $\mathcal{O}(\alpha_s y^2 + y^4)$ correction less than 10%. On the other hand, the dependence of the

$\mathcal{O}(\alpha_s y^2 + y^4)$ correction on m_0 becomes essential for relatively small $m_{1/2}$ ($m_{1/2} \approx 200$ GeV).

It should be noted that in the considered range of the CMSSM parameter space the $\mathcal{O}(\alpha_s^2 + \alpha_s y^2 + y^4)$ correction is positive.

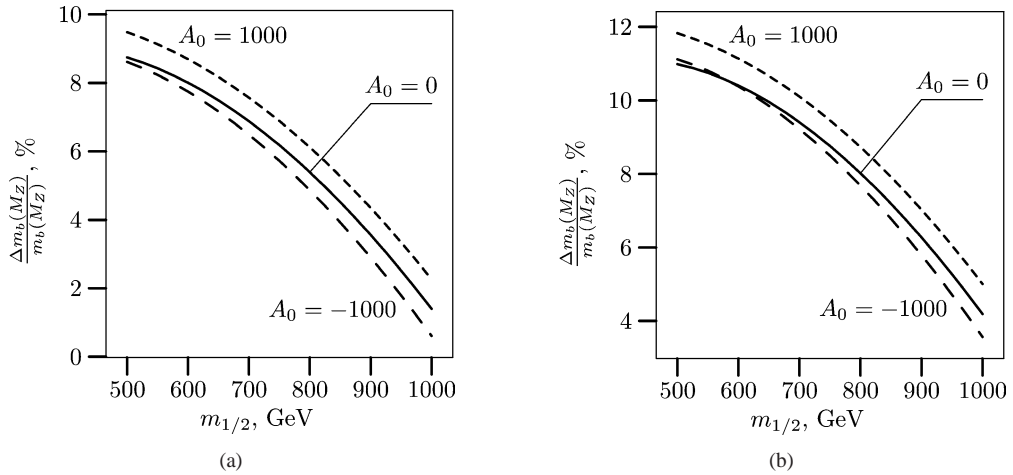


Fig. 5. A_0 dependence of the $\mathcal{O}(\alpha_s^2 + y^4 + \alpha_s y^2)$ two-loop correction to $\Delta m_b(M_Z)/m_b(M_Z)$. Here $m_0 = 1000$ GeV. Varying A_0 from -1000 to 1000 GeV does not change the behaviour of the correction significantly. (a) $\tan \beta = 35$; (b) $\tan \beta = 50$.

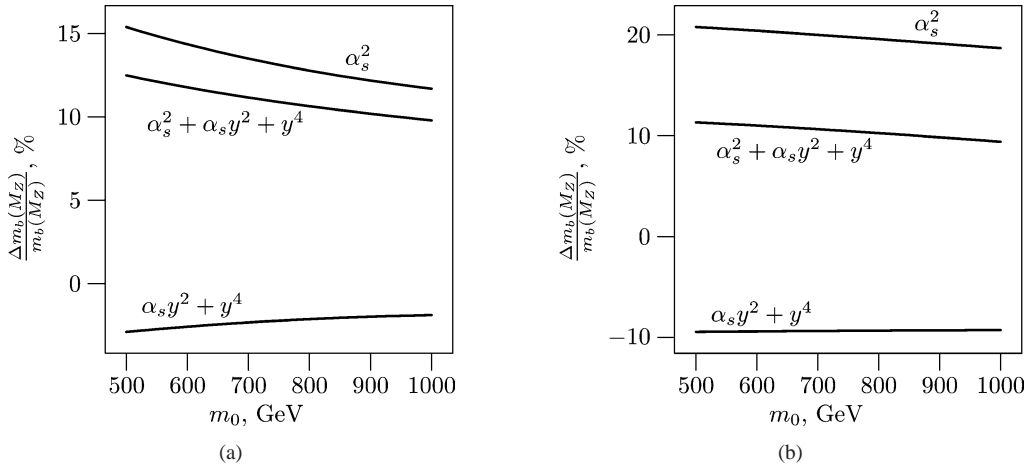


Fig. 6. Different two-loop MSSM corrections to $\Delta m_b(M_Z)/m_b(M_Z)$ as a function of m_0 . Here $A_0 = 0$, $\tan \beta = 50$. Dependence of the $\mathcal{O}(\alpha_s y^2 + y^4)$ correction on m_0 is essential for $m_{1/2} = 200$ GeV. On the other hand, for $m_{1/2} = 700$ GeV the change of m_0 from 400 to 1000 GeV yields the change of the $\mathcal{O}(\alpha_s y^2 + y^4)$ correction less than 10%. (a) $m_{1/2} = 200$ GeV; (b) $m_{1/2} = 700$ GeV.

4. Conclusion

In this Letter, we presented the results of calculation of the two-loop corrections to the relation between pole and running mass of the b -quark which are proportional to the Yukawa couplings of heavy quarks (y_t, y_b). We provided a numerical analysis of the value of these corrections in different regions of the CMSSM parameter space. The corrections presented here can be used in a renormalization group analysis of the Yukawa coupling unification. It was

also found that m_b corrections significantly affect low-energy phenomenology where the b -quark enters [22]. The analysis given in [23] showed that the neutralino relic density is very sensitive to the mass of the b -quark for large $\tan \beta$, thus the result considered in this Letter may have important implications to the dark matter searches. We are going to study these issues and calculate more $\mathcal{O}(\alpha_s^2)$ terms in the large mass expansion of the relation between the top pole and $\overline{\text{DR}}$ masses, which requires some improvement of our code.

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